

Can the Reissner-Nordström black hole or Schwarzschild black hole be the stable Planck-scale particle accelerator?

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Abstract

It is shown that the extremal Reissner-Nordström black hole, the non-extremal one with multiple scattering particles, and the Schwarzschild black hole with radial head-on particles are stable under the collision of the particles near the horizon, if the back-reaction effect and the effect generated by gravity of particles are involved. Moreover, the collision near Reissner-Nordström black holes with astrophysically typical mass can not generate the Planck-scale center-of-mass energy. However, the head-on collision near the typical primordial black hole could just occur at the Planck-energy scale.

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I. INTRODUCTION

Recently, Bañados, Silk and West (BSW) [1] proposed a mechanism to obtain arbitrary high center-of-mass (CM) energy of two particles collided on the horizon of an extremal Kerr black hole (BH), which was hence asserted as a natural Planck-scale particle accelerator and the possible origin for the very highly energetic astrophysical phenomena. However, the authors of [2] and [3] pointed out that some factors, such as the astrophysical limit of maximal spin of Kerr BHs, would inhibit the Planck-scale collision.

To avoid the astrophysical limitation of Kerr BHs, several methods have been presented. The first is the called multiple scattering mechanism, by which the BSW process still can be used to get arbitrary high CM energy of colliding particles near the nonextremal Kerr BH [4]. Another more direct idea is to consider different extreme rotating BHs [5]. Actually, Zaslavskii showed the unbound energy of colliding particles on the general rotating extreme horizon or the nonextremal horizon considering multiple scattering [6]. Noticing the alternative options for generating extremal black holes, the BSW mechanism has been further studied by calculating the escaping flux of massless particles for maximally rotating black holes, and it was suggested that the received spectrum should typically contain signatures of highly energetic products [7], see also the sequent numerical estimation [8].

Most works [9–16] on the BSW mechanism necessitate the rotation of BHs. But in [17], it was proposed that a non-rotating but charged Reissner-Nordström (RN) BH can also serve as an accelerator with arbitrarily high CM energy of charged particles collided at the extreme horizon or nonextremal horizon considering the multiple scattering. In particular, it was demonstrated that the upper bound of the electric charge of BHs after Schwinger emission is large enough to allow the ultra-high CM energy of charged colliding particles, provided that the nonextremal BH is not too light ($> 10^{20}g$). Furthermore, the general stationary charged BH was studied in detail [18], suggesting that the potential acceleration to large energies should be taken seriously as the manifestation of general properties of rotating or charged BHs. Other general kinematic explanation was presented in [19].

All the aforementioned works only considered the collision between ingoing particles. The collision between ingoing and outgoing particles has been studied in [20] very early, and recently reintroduced more generally in [21], where it was shown that the unlimited CM energy can also be attained. Compared with the BSW process that requires one particle having

critical angular momentum or charge, this divergence can be seen as a direct consequence of infinite redshift near the horizon.

However, beside the astrophysical limit of Kerr BHs, there are other factors to prohibit the ultra-energetic collisions near extreme Kerr BHs, such as the gravitational radiation [2], the back-reaction effect [2], and the effect of gravity generated by colliding particles [22]. Here the called back-reaction effect is considered as due to absorbing the first pair of colliding particles, then the extremal Kerr BH becomes non-extremal and the CM energy declines rapidly. The effect of gravity of particles seems similar to the back-reaction effect in its physical significance, but is tackled more rigorously in the case of the collision of two spherical dust shells in the neighborhood of an extreme RN BH. It was shown [22] that an upper limit exists for the total energy of colliding shells in the CM frame in the observable domain due the gravity of shells. Since the similarity between the BSW process for RN BHs and Kerr BHs, it was argued that the upper limit also exists for extreme Kerr BHs.

In this paper, we will take into account the back-reaction effect and the effect arising from the gravity of particles (shells) for the collision of particles near the extremal RN BH, the collision near the non-extremal RN BH considering multiple scattering, and the radial head-on collision near the Schwarzschild BH. The gravitational radiation can be neglected since these cases are spherically symmetric. We will focus on two important problems: whether or not the CM energy of colliding particles can reach the Planck scale and even the BH mass. Note that the later problem means whether the BH is stable against minor perturbations induced by dropping small particles into the black hole. This new paradoxical instable mechanism of BHs under the BSW process was suggested by Lake [23] and clearly presented in [22], where the extremal RN BH was found to be stable if the gravity of particles is incorporated. We will study this instability for other mentioned cases.

II. BACK-REACTION EFFECTS

We set out to investigate back-reaction effects. The metric for RN spacetimes is

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$

The event horizon is located at $r_H = M + \sqrt{M^2 - Q^2}$. Considering a test particle with charge q and energy E per unit rest mass m , the radial timelike geodesics is

$$u^t \equiv \frac{dt}{d\tau} = \frac{1}{f} \left(E - \frac{qQ}{r} \right), \quad u^r \equiv \frac{dr}{d\tau} = \pm \sqrt{\left(E - \frac{qQ}{r} \right)^2 - f}, \quad (2)$$

where the positive and minus signs of u^r correspond to outgoing and ingoing geodesics, respectively. The CM energy can be obtained by

$$E_{c.m.}^2 = -(m_1 u_1 + m_2 u_2)^2, \quad (3)$$

where index $i = 1, 2$ denote two different particles.

A. Extremal RN BHs

For two test particles with the equal mass m_0 , charges q_i ($i = 1, 2$), and energy parameters E_i per unit rest mass, the CM energy (3) can be expressed as [17]

$$\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{1}{2} \left(\frac{E_2 r_H - Q q_2}{E_1 r_H - Q q_1} + \frac{E_1 r_H - Q q_1}{E_2 r_H - Q q_2} \right) \quad (4)$$

Consider extremal RN BHs with $Q = M$ and one ingoing particle with critical charge $q_1 = E_1$, which is capable of touching the extremal horizon from infinity. It is easy to notice that the CM energy could be divergent when $q_1 = E_1$ and $q_2 \neq E_2$, which implies that the collision on the extremal horizon of RN BHs might produce arbitrary high CM energy, and seems to destroy the BH background. We note that the angular momentum of particles will not lead to the qualitative difference.

Now we will tackle the back-reaction effect following the spirit in Ref. [2], but not using the dimensionless parameters of BHs. Upon absorption of the first pair of colliding particles, the new charge, mass and horizon of BHs are (see Fig. 1)

$$Q' = Q + m_0(q_1 + q_2), \quad M' = M + m_0(E_1 + E_2), \quad r'_H = M' + \sqrt{M'^2 - Q'^2}. \quad (5)$$

Since $\sigma = \frac{m_0}{M} \ll 1$, the CM energy (4) after the first collusion can be obtained as

$$\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{E_2 - q_2}{2E_1 \sqrt{2E_1 + 4E_2 - 2q_2}} \frac{1}{\sqrt{\sigma}} + \mathcal{O}(\sigma)^0. \quad (6)$$

Focusing on the order of magnitude of the CM energy, Eq. (6) can be recast as

$$E_{c.m.} \lesssim m_0^{\frac{3}{4}} M^{\frac{1}{4}} \simeq 10^{12} \left(\frac{m_0}{1 \text{ MeV}} \right)^{\frac{3}{4}} \left(\frac{M}{100 M_\odot} \right)^{\frac{1}{4}} \text{ GeV}. \quad (7)$$

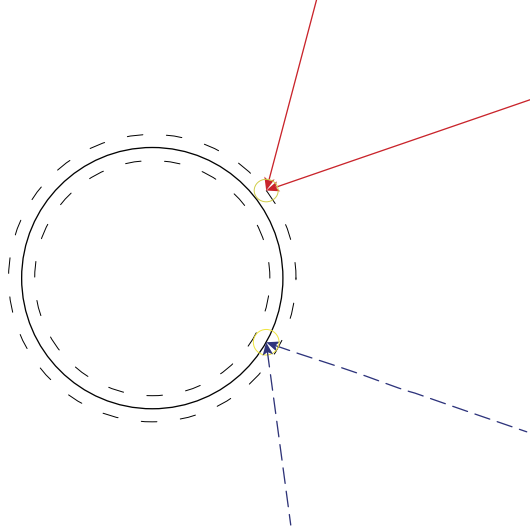


FIG. 1: Schematic diagram of the back-reaction effect of extremal RN BHs. The blue dashed lines represent the first pair of colliding particles. They collide on the horizon of extremal RN BHs, which is denoted by the black solid circle. After absorbing the first pair of colliding particles, the degenerated horizon is split into the event horizon and Cauchy horizon which are represented by the black dashed circles. The second pair of colliding particles are represented by the red solid lines, they collide on the event horizon.

We note that Eq. (7) is same as the order of magnitude of CM energy obtained in [2] for extremal Kerr BHs. Thus, one can have the similar result, namely, the extremal BHs are stable under the particle collision as long as the mass of the particles is smaller than the mass of the BHs. In addition, the Planck energy is hard to be attained for typical values of parameters. For instance, for the BH with $M \sim 100M_{\odot}$, a hypothetical dark matter particle would need a mass $m_0 \sim 10^{10}MeV$ to allow for more than one Planck-scale event. The collision of a single electron pair would reduce the charge of a $100M_{\odot}$ BH sufficiently to inhibit any further Planck-scale collision.

B. Non-extremal RN BHs

We have considered the extremal RN BHs with the mass of typical astrophysical BHs $M \sim 100M_{\odot}$, which is only of academic interest since the astrophysical BHs can not possess a large charge and should be non-extremal. However, in the non-extremal RN background, a particle with the critical charge $q_1 = E_1 r_H / Q$ can not arrive at the event horizon from

infinity [17, 18]. Similar situation happens in non-extremal Kerr BHs where the particle with critical angular momentum can not reach the horizon. However, noticing that there exists some region close to the horizon where one has particles moving with angular momentum arbitrary close to the critical value, Pavlov [4] presented that the BSW process is also applicable to non-extremal Kerr BHs by invoking the multiple scattering mechanism to amplify the angular momentum of the falling particle from infinity. This mechanism has been used to RN BHs [17] and more general charged background [18] where the small charge of particle from infinity can be amplified near the horizon. The multiple scattering mechanism in charged BHs is effective since a particle with the charge close to the critical value $q_1 = (1 - \delta)E_1 r_H / Q$, which is generated by the multiple scattering near the horizon, may exist near the event horizon [17, 18]. The CM energy of the collision between a near critical particle and another particle with charge q_2 on the event horizon is

$$\frac{E_{c.m.}^2}{2m_0^2} = \frac{E_2 r_H - Q q_2}{2E_1 r_H} \frac{1}{\delta} + \mathcal{O}(\delta). \quad (8)$$

One can find that the CM energy will be divergent while $\delta \rightarrow 0$.

Now, let us count the back-reaction effect. Due to absorbing the first pair of colliding particles, the parameters of BHs become to Eq. (5), which are similar to the case of extremal RN BHs. However, because the multiple scattering mechanism is still applicable for the following collisions to provide the particle with new near critical charge $q'_1 = (1 - \delta)E_1 r'_H / Q'$, the corresponding CM energy will be the same as Eq. (8), except to replace r_H by r'_H . Thus, the CM energy of multiple scattering particles colliding near the non-extremal RN BH will still be divergent after involving the back-reaction effect. We also note that, by a similar consideration for the non-extremal Kerr BHs with multiple scattering particles, one can obtain the same conclusion. Thus, if one would like to guarantee the stability of BHs under the BSW process considering multiple scattering, as it should be, other effects need to be considered.

C. Radial head-on collision in Schwarzschild spacetimes

The authors of Ref. [1] showed that one could not get ultra-high CM energy by the collision of two ingoing particles in Schwarzschild spacetimes. However, Zaslavskii noticed that [21] the unbound CM energy can be achieved by the collision at horizon between

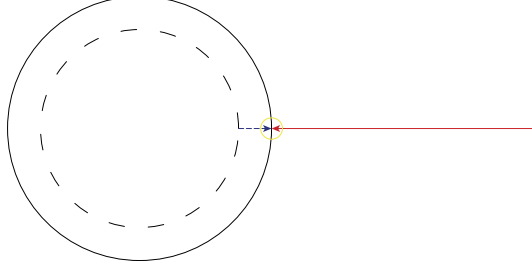


FIG. 2: Schematic diagram of the back-reaction effect for the radially head-on collision in Schwarzschild spacetimes. The solid circle represents the location of horizon before the collision. It shrinks back to the dashed circle after emitting a particle, which is represented by the blue dashed line. The outgoing particle collides with an ingoing particle, which is represented by the red solid line, on the location of the original horizon.

outgoing particle and ingoing particle. In this subsection, we will calculate the back-reaction effect for the radial head-on collision. Here, the back-reaction effect has somewhat of an inverse relationship to the one discussed above, where it is generated by the BH absorbing the colliding particles. Now the back-reaction effect is considered as arising from the shrink of the horizon due to emitting an outgoing particle by BHs.

According to Eqs. (2) and (3) with $Q = q_i = 0$, one can obtain the CM energy of the collision between an outgoing particle and an ingoing particle in Schwarzschild BHs, which is

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2m_1m_2 \frac{E_1E_2r + \sqrt{(E_1^2r - r + 2M)(E_2^2r - r + 2M)}}{r - 2M}. \quad (9)$$

When the collision occurs on the horizon, i.e. $r \rightarrow 2M$, the denominator of Eq. (9) equates to zero and the numerator is nonvanishing. Hence, this CM energy of the collision is divergent on the horizon.

To take the back-reaction effect into account, we will assume that the original mass of BH is $M + E_2m_2$ and the horizon is $r_H = 2(M + E_2m_2)$. After the BH emits a particle with mass m_2 and energy E_2 , the horizon will be changed to $r_H = 2M$ and the collision should occur at $r \geq 2(M + E_2m_2)$, see Fig. 2. So the CM energy (9) is less than that of the collision at $r = 2(M + E_2m_2)$:

$$E_{c.m.}^2 = \frac{4m_1m_2E_1E_2}{\sigma} + \mathcal{O}(\sigma)^0 = 4E_1E_2m_1M + \mathcal{O}(\sigma)^0 \quad (10)$$

where $\sigma = m_2/M$. From this equation, we can estimate the order of magnitude for the CM

energy as following:

$$E_{c.m.} \lesssim m_1^{\frac{1}{2}} M^{\frac{1}{2}} \simeq 10^{28} \left(\frac{m_1}{1MeV} \right)^{\frac{1}{2}} \left(\frac{M}{100M_{\odot}} \right)^{\frac{1}{2}} GeV. \quad (11)$$

Several remarks are in order. First, the colliding CM energy does not relate to the rest mass of outgoing particles. This suggests that Eq. (11) is general even for the collision between an outgoing photon and an ingoing massive particle. Actually, one can check it following the similar process, although the geodesics of photons are different to massive particles. Second, similar to the previous case of extremal RN BHs, the colliding CM energy can not reach the order of magnitude of BH's mass, since it needs the rest mass of ingoing particle $m_1 \sim M$. Thus, the Schwarzschild BH is stable under the collision of outgoing and ingoing particles. Third, the CM energy is proportional to $M^{1/2}$, different from $M^{1/4}$ in Eq. (7). This is an essential difference. Consequently, for a Schwarzschild BH with the 100 solar mass and $m_1 \sim 1MeV$, the CM energy is $E_{c.m.} \sim 10^{28}GeV$, which is higher than the Planck energy. Finally, let us discuss how to set an out-going particle near the horizon. One might consider a particle which starts at infinity and falls into the horizon. In principle, it is possible to arrange that the particle decays into two particles near the horizon, where one particle falls into the horizon and the other goes out. The process is similar to the Penrose process with an additional restriction that the split just occurs outside the event horizon. So it would be quite rare in astrophysics. To be more realistic, we would like to invoke the mechanism of Hawking radiation. Consider vacuum fluctuations which create a particle-antiparticle pair in the vicinity of the horizon of a black hole. If the region is sufficiently small (that is just necessary for the collision with high CM energy), an antiparticle with negative energy can be created inside the horizon and an escaping particle outside with finite positive energy. However, the Hawking temperature for Schwarzschild BHs is $T \sim 10^{-7} \frac{M_{\odot}}{M} K$ which is very low for astrophysically large BHs and they would evaporate completely on a very long time scale $\tau \sim 10^{64} \left(\frac{M}{M_{\odot}} \right)^3$ yr. Thus, we are turned to the small primordial black holes with the typical mass $M \sim 10^{15}g \sim 10^{-18}M_{\odot}$, which are hot enough ($\sim 10^{11}K$) to radiate a significant fraction of their mass over the 14-billion-year age of the universe and would be exploding today [24]. Interestingly, one can find that the small mass of such primordial black holes can just afford the Planck-scale collision.

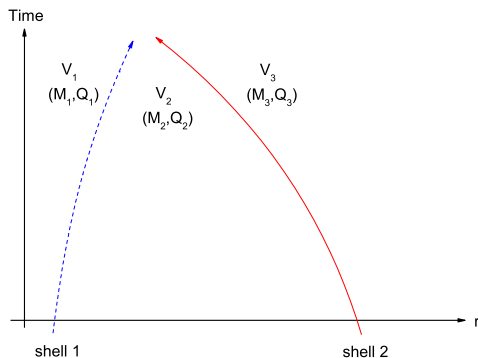


FIG. 3: Schematic spacetime diagram of two spherical timelike shells. Two shells divides the spacetime into three parts. Shell 1 (blue dash line) is outgoing and shell 2 (red solid line) is ingoing. For the case with two ingoing shells, see Fig. 2 in Ref. [22].

III. THE GRAVITY GENERATED BY COLLIDING SHELLS

We will investigate the effect of the gravity generated by colliding particles. The approximate method of evaluating this effect is to consider the collision of two infinitesimal thin spherical shells. In Ref. [22], it has been showed that the CM energy of this collision in the extremal RN background can not reach the Plank energy respecting this effect. Here, we would like to study the colliding energy for infinitesimally thin spherical shells in nonextremal RN spacetimes and the radial head-on collision in Schwarzschild spacetimes.

The existence of spherical shells divides the spacetime into three regions, as show in Fig. 3, where we denote them with the index $i = 1, 2, 3$. According to the Birkhoff's theorem, the metrics of those spacetimes can be described by

$$ds^2 = -f_i dt_i^2 + f_i^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (12)$$

where $f_i = 1 - \frac{2M_i}{r} + \frac{Q_i^2}{r^2}$, no matter the spherical objects are radially collapsing or exploding. Note that all coordinates except for the time t are common to different regions. The BH horizons in different regions are located at $r_i = M_i + \sqrt{M_i^2 - Q_i^2}$.

A. RN spacetimes

Consider that the shell 1 (inner shell) is a charged dust shell with charge $q_1 = Q_2 - Q_1$ and $\epsilon\mu = M_2 - M_1$, where μ is the proper mass of the shell and ϵ is the specific energy; the

shell 2 (outer shell) is composed of neutral dust and has the same proper mass and specific energy as the shell 1. Using Israel junction conditions [25], the effective potential of shell 1 has been obtained in [22]:

$$V_{eff}^1 = -(u_1^r)^2,$$

where the radial component of 4-velocity is written in the form

$$u_1^r = -\sqrt{-1 - \frac{q_1^2}{4r^2} - \frac{\langle Q \rangle_1^2}{r^2} + \frac{2\langle M \rangle_1}{r} + \left(\epsilon - \frac{q_1 \langle Q \rangle_1}{r\mu} \right)^2 + \frac{\mu^2}{4r^2}}, \quad (13)$$

with

$$\langle M \rangle_1 = \frac{M_2 + M_1}{2}, \quad \langle Q \rangle_1 = \frac{Q_2 + Q_1}{2}. \quad (14)$$

For shell 2, the radial velocity has the same form except to replace q_1 as zero and take $\langle M \rangle_1$ and $\langle Q \rangle_1$ as

$$\langle M \rangle_2 = \frac{M_3 + M_2}{2}, \quad \langle Q \rangle_2 = \frac{Q_3 + Q_2}{2}. \quad (15)$$

Introducing the “center of mass frame” at the collision event of the shells, the CM energy of the colliding shells has been given by [22]

$$\frac{E_{c.m.}^2}{2\mu^2} = 1 - g_{ab}u_1^a u_2^b, \quad (16)$$

which can be rewritten as

$$\frac{E_{c.m.}^2}{2\mu^2} = 1 - \frac{u_1^r u_2^r}{f_2} + \sqrt{\left[1 + \frac{(u_1^r)^2}{f_2} \right] \left[1 + \frac{(u_2^r)^2}{f_2} \right]}. \quad (17)$$

Eq. (17) can be expanded as

$$\frac{E_{c.m.}^2}{2\mu^2} = \frac{(u_1^r + u_2^r)^2}{2u_1^r u_2^r} + \mathcal{O}(f_2)^1. \quad (18)$$

At first glance, the CM energy could be divergent when $f_2 = 0$, $(u_1^r)^2 = 0$ and $(u_2^r)^2 \neq 0$. However, noticing that r_3 , i.e. the event horizon in region 3, is larger than the r_2 , distant observers could not see the collision on r_2 . Consequently, the observable CM energy is less than that of the collision at r_3 .

Now let us calculate the observable maximized CM energy. Since $\sigma = \mu/M_1$ is very small, $f_2(r_3)$ is close to $f_2(r_2) = 0$. Then Eq. (18) at r_3 is still effective in general, except for the case of

$$[u_1^r(r_3)]^2 = 0, \quad [u_2^r(r_3)]^2 \neq 0. \quad (19)$$

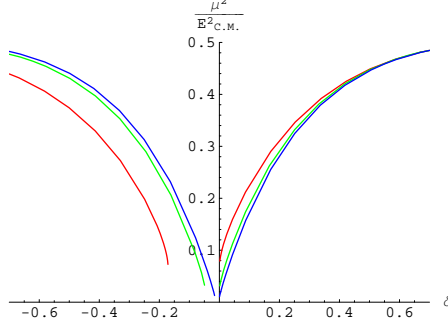


FIG. 4: The inverse of the observable CM energy $2\mu^2/E_{c.m.}^2$ with respect to the charge of inner shell $q_1 = q_c(1 - \delta)$. The charge of BHs is assumed as $x = 0.5$ and specific energy $\epsilon = 2$. The red, green, and blue lines (from top to bottom in the region $\delta > 0$) denote $\sigma = 10^{-2}$, 10^{-3} , 10^{-4} , respectively. This figure indicates q_c approaching the critical charge for the observable maximized CM energy when $\sigma \rightarrow 0$.

Eq. (19) means

$$q_1 = q_c \equiv Q_2 - \sqrt{Q_2^2 - 2r_3\epsilon\mu + \mu^2 - 2\mu\sqrt{Q_2^2 + r_3(r_3 - 2M_1 - 2\epsilon\mu)}}. \quad (20)$$

Note $[u_2^r(r_3)]^2 \neq 0$ since

$$[u_2^r(r_3, q_c)]^2 = \epsilon^2 + \mathcal{O}(\sigma)^1.$$

An important observation is that the divergent Eq. (18) suggests q_c approaching the critical charge for the observable maximized CM energy when $f_2(r_3)$ approaches zero. In other word, we can treat q_c as the critical charge since σ is very small. To confirm that the derivation of critical charge is rigorous, we have checked it using numerical method, see Fig. 4 for Eq. (17) at r_3 with respect to q_1 . Note that we have set $Q_2 = M_2(1 - x)$ ($0 \leq x \leq 1$) and $q_1 = q_c(1 - \delta)$ ($\delta \leq 1$) to characterize the deviation.

Furthermore, taking $r = r_3(1 + y)$ ($y \geq 0$), one can find that the shell 1 with q_c can not touch the r_3 from infinity since the effective potential with $q_1 = q_c$ is

$$V_{eff}^1 = \frac{2\sqrt{x(2-x)} \left[1 - \sqrt{x(2-x)} \right]}{(1-x)^2} y + \mathcal{O}(y)^2 + \mathcal{O}(\sigma)^{\frac{1}{2}} > 0.$$

However, following the spirit in [4], one might consider that the shell 1 is generated by the multiple scattering near r_3 with near critical charge $q_1 = q_c(1 - \delta)$, since the effective potential is negative at r_3 :

$$V_{eff}^1 = -4\delta^2 + \mathcal{O}(\delta)^3 + \mathcal{O}(\sigma)^{\frac{1}{2}} < 0. \quad (21)$$

Moreover, one should be noticed that in order that the outer shell overtakes the inner shell, $[u_2^r(r_3)]^2 - [u_1^r(r_3)]^2$ should be positive, which can be seen from

$$[u_2^r(r_3)]^2 - [u_1^r(r_3)]^2 = \epsilon^2 + \mathcal{O}(\sigma)^{\frac{1}{2}} + \mathcal{O}(\delta)^1.$$

Thus, the maximized CM energy can be evaluated from Eq. (17) with critical charge $q_1 = q_c(1 - \delta)$ at r_3 :

$$\frac{E_{c.m.}^2}{2\mu^2} = \sqrt{\frac{\epsilon}{2}} \sqrt{1 + \sqrt{x(2-x)}} \frac{1}{\sqrt{\sigma}} + \mathcal{O}(\sigma)^0 + \mathcal{O}(\delta)^{\frac{1}{2}}. \quad (22)$$

Since Eq. (22) has the same order of magnitude of Eq. (6), we know that the CM energy declines rapidly upon counting the gravity of shells, and it can not reach the Planck scale.

We note that when $x = 0$ denoting region 2 as an extremal RN spacetime, Eq. (22) reduces to

$$\frac{E_{c.m.}^2}{2\mu^2} = \sqrt{\frac{\epsilon}{2}} \frac{1}{\sqrt{\sigma}} + \mathcal{O}(\sigma)^0,$$

which is a little larger than the result obtained in [22]

$$\frac{E_{c.m.}^2}{2\mu^2} = \sqrt{\frac{\epsilon}{2}} (\epsilon - \sqrt{\epsilon^2 - 1}) \frac{1}{\sqrt{\sigma}} + \mathcal{O}(\sigma)^0,$$

where the involved critical charge q_c is not exactly solved from $[u_1^r(r_3)]^2 = 0$ but from $[u_1^r(r_2)]^2 = 0$.

B. The head-on collision

For the radial head-on collision in Schwarzschild spacetimes, equations of motion for the shells are described by Eqs. (13), (14) and (15) with $q_i = Q_i = 0$, but the radial component of 4-velocity of inner shell should add a minus sign. Thus, one can calculate the CM energy of the head-on collision of shells from Eq. (17), which is given by

$$\frac{E_{c.m.}^2}{2\mu^2} = \frac{-1}{4r(r-r_2)} \left[8M_1r - 4r^2(\epsilon^2 + 1) + 8r\epsilon\mu + \mu^2 - \sqrt{8M_1r + 4r^2(\epsilon^2 - 1) + 12r\epsilon\mu + \mu^2} \sqrt{8M_1r + [2r(\epsilon - 1) + \mu][2r(\epsilon + 1) + \mu]} \right].$$

It is divergent when $r = r_2$. However, the observable CM energy should be less than that of the collision at r_3 , which is

$$\frac{E_{c.m.}^2}{2\mu^2} = \frac{2\epsilon}{\sigma} + \mathcal{O}(\sigma)^0.$$

This equation is similar to Eq. (10), indicating that a Schwarzschild BH could be a stable Planck-scale particle accelerator when gravity of shells is involved.

IV. CONCLUSION

In this work, we show that the BSW process involving the back-reaction effect can not produce the Planck-scale CM energy in the background of the extremal RN BH with astrophysically typical mass, but for the non-extremal RN BH, the CM energy of colliding particles generated by multiple scattering still can be arbitrary high. However, if the effect of the gravity of particles (shells) is incorporated, even the Planck energy can not be attained. This case demonstrates that the back-reaction effect due to absorbing the first pair of colliding particles and the effect generated by gravity of particles sometimes are different operationally, although their physical significance seems similar. Moreover, it is found that the Schwarzschild BH is stable under the radial head-on collisions if any effect is involved. Interestingly, for small primordial BHs with the typical mass $10^{15}g$, the head-on collision could just occur at the Planck-energy scale. Whether this result might have the astrophysically observable phenomenon is very deserved to be investigated in the future.

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